

On the metal–insulator transition in the two–chain model of correlated fermions

H. J. Schulz

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Laboratoire de Physique des Solides, Université Paris–Sud, 91405 Orsay, France

Abstract

The doping–induced metal–insulator transition in two–chain systems of correlated fermions is studied using a solvable limit of the $t - J$ model and the fact that various strong- and weak-coupling limits of the two-chain model are in the same phase, i.e. have the same low-energy properties. It is shown that the Luttinger–liquid parameter K_ρ takes the universal value unity as the insulating state (half-filling) is approached, implying dominant d-type superconducting fluctuations, independently of the interaction strength. The crossover to insulating behavior of correlations as the transition is approached is discussed.

Models of two parallel chains of correlated fermions are of interest for the understanding of the physical properties of a number of systems: (i) two-leg spin ladder systems of the type $\text{Sr}_x\text{Ca}_{14-x}\text{Cu}_{24}\text{O}_{41}$ become conducting and even superconducting under pressure,[1] even though they are insulating under ambient conditions;[2] (ii) the electronic structure of certain types of carbon nanotubes[3] is described by very similar models;[4] (iii) two-channel quantum wires may also show interesting interaction effects. The two-chain case, being much easier to treat by controlled analytical and numerical methods than genuinely two- or three-dimensional models, is also helpful in understanding long-standing questions about the existence of superconductivity in models of correlated fermions.

Theoretical work on the two-chain model has either considered situations well away from half-filling,[5, 6, 7] or concerned the half-filled (one electron per site) case where unklapp scattering leads to a Mott insulator state with a spin gap.[8, 9, 7, 10] However, very little work [10, 11] concerned the close vicinity of half-filling, a situation of considerable interest, in particular in view of the physics of doped spin ladders. In the present paper I wish to show that in that situation for quite general interaction strengths the Luttinger liquid parameter K_ρ becomes unity (the same as for noninteracting fermions) and that consequently one can expect a “d-wave” superconducting state. This is to be contrasted with the single-chain case where close to half-filling one finds the universal strong coupling value $K_\rho = 1/2$ and dominant antiferromagnetic correlations.[12, 13, 14]

Numerical work has shown strong indications of superconductivity in Hubbard and $t-J$ ladders, both calculating correlation functions directly[15, 16, 17, 18, 19] and by exploiting the periodicity of the ground state energy in the presence of a magnetic flux.[20] Nevertheless, numerical results are very hard to interpret in the vicinity of half-filling and numerical work has thus not helped much in understanding the doping-induced metal-insulator transition.

I will now summarize the current understanding of the properties of two chains of interacting fermions coupled by interchain hopping and well away from half-filling. In the case of weak interactions a combination of perturbative renormalization group calculations and bosonization [5, 6, 7] has shown the existence of a gap in the spin excitation spectrum and that superconducting correlations of d type dominate. More precisely, define the superconducting order parameter as

$$\Delta_r = \frac{1}{\sqrt{2}} (a_{1\uparrow r} a_{2\downarrow r} - a_{1\downarrow r} a_{2\uparrow r}). \quad (1)$$

Here r labels sites along the chains, and a_{isr} is the fermion annihilation operator on site r , chain i , and with spin projection s . It is appropriate to call this order parameter “d-wave” because in Fourier space components with transverse wavevector 0 and π have opposite sign. Correlations of this order parameter decay slowly with distance:

$$\langle \Delta_r^+ \Delta_0 \rangle \approx r^{-\eta_{SCd}}, \quad (2)$$

with $\eta_{SCd} = 1/(2K_\rho)$ and K_ρ a number close to unity. Powerlaw decay of correlations also exists for the $4k_F$ component of the particle density:

$$\langle n_r n_0 \rangle \approx \cos(4k_F r) r^{-\eta_{4k_F}}, \quad (3)$$

and one has the *scaling relation*

$$\eta_{SCd} \eta_{4k_F} = 1 \quad (4)$$

(one has $k_F = \pi n/2$, where n is the average number of fermions per site). All other correlation functions, in particular those representing magnetic ordering, decay exponentially with distance.

Remarkably, analogous results can also be obtained for some strong-coupling cases:[6] first, if correlations within a single chain are so strong that $K_\rho < 1/3$, renormalization-group generated interchain interactions dominate over the single particle hopping, and the resulting problem can be solved, leading to the same powerlaws (2,3) and the same scaling law (4) as in the weak coupling case. However, because now $K_\rho < 1/3$, the $4k_F$ CDW correlations dominate over d-wave superconductivity. Secondly, a particular limit of the two-chain $t - J$ model, to be discussed in more detail below, also leads to the same powerlaws and the same scaling relation. The natural conclusion from these findings in three different limits is that *the correlated two-chain model is in the same phase, characterized by eqs.(2,3,4) in a large region of interaction strength, both for weak and for strong correlation.*

I now wish to determine the value of the Luttinger liquid parameter K_ρ in the vicinity of the doping-induced metal-insulator transition, i.e. for $n \rightarrow 1$ but $n \neq 1$. The direct use of the renormalization group approach is highly impractical because in this case umklapp interactions play an important role at high energies, but drop out at low energies. At intermediate energies, one then passes through a strong-coupling region which is impossible to treat systematically. The “ $K_\rho < 1/3$ ” approach is equally impossible because for $n \rightarrow 1$ in a single chain one universally has $K_\rho \rightarrow 1/2$, [12, 13, 14] i.e. one drops outside the validity range of this approach. However, the $t - J$ model gives us the possibility to reach some exact and general conclusions. To be specific I consider the spatially anisotropic “ $t - J - J_\perp$ model” [18] with Hamiltonian

$$\begin{aligned} H = & -t \sum_{i,r} (a_{isr}^\dagger a_{isr+1} + H.c.) - t_\perp \sum_r (a_{1sr}^\dagger a_{2sr} + H.c.) \\ & + J \sum_{i,r} \mathbf{S}_{ir} \cdot \mathbf{S}_{ir+1} + J_\perp \sum_r \mathbf{S}_{1r} \cdot \mathbf{S}_{2r} \quad , \end{aligned} \quad (5)$$

where $\mathbf{S}_{ir} = a_{isr}^\dagger a_{is'r}(\boldsymbol{\sigma})_{ss'}/2$ is the spin operator on site (i, r) and the usual no-double-occupancy constraint is imposed. J and J_\perp are the exchange constants along and perpendicular to the ladder, and t and t_\perp the corresponding hopping integrals.

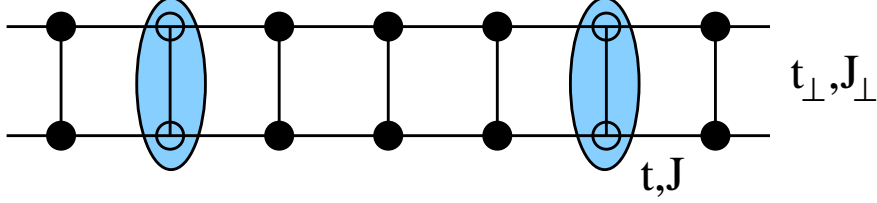


Figure 1: Low-energy Hilbert space of the $t - J - J_\perp$ model in the limit $J_\perp \gg t, t_\perp, J$: rungs are either doubly occupied and in a singlet state (full dots), or both sites are empty (shaded ellipses).

In the limit where $J_\perp \gg J, t, t_\perp$ analytical progress can be made:[18] then the low-energy Hilbert space consists of rungs of the ladder where either both sites are occupied and form a singlet or both sites are empty (fig.1). Singly occupied sites or rungs with a triplet lie higher by an energy of order J_\perp . It is then convenient to consider the state where all rungs are occupied by singlets as the physical vacuum and creation of an empty rung as creation of a boson on site r , with associated boson creation operator $b_r^+ = \Delta_r$. In second order perturbation theory in t and J one then obtains an effective Hamiltonian for the low-energy Hilbert space:

$$H_{\text{eff}} = -t_{\text{eff}} \sum_r (b_r^+ b_{r+1} + H.c.) + V_{\text{eff}} \sum_r n_r n_{r+1} \quad , \quad (6)$$

where $t_{\text{eff}} = 8t^2/3J_\perp$, $V_{\text{eff}} = (16t^2/3 - 3J^2/8)/J_\perp$, $n_r = b_r^+ b_r$, and now there is the “hardcore constraint” $n_r \leq 1$. Because of that constraint, one can straightforwardly transform the model into a spin-1/2 language by setting $S_r^+ = b_r^+$ and $S_r^z = n_r - 1/2$, to recover the well-known spin-1/2 XXZ spin chain.[21, 22] In Haldane’s paper the interesting Luttinger liquid parameter K_ρ is determined. In particular, if the original fermionic model is close to the insulating state at half-filling, $n \rightarrow 1$, corresponding to a very dilute hardcore boson gas, one has $K_\rho = 1 + O(1 - n)$, independent of the value of the interaction, i.e. *K_ρ takes a universal value when the metal-insulator transition is approached.* Moreover, I have argued above that both the weak-interaction limit of the two-chain model and the strongly interacting chains weakly coupled by interchain hopping are in the same phase as the $t - J - J_\perp$ model. One thus concludes that upon approaching the doping induced metal-insulator transition, $n \rightarrow 1$, the Luttinger liquid parameter K_ρ takes the universal value unity, independently on whether correlations are weak or strong. Via the relations (2,3,4) this then implies that *d -type superconducting correlations are strongly dominant close to the metal-insulator transition.*

It is interesting to understand how the powerlaws (2,3) connect to the behavior expected for the insulating case. Consider first the $4k_F$ CDW correlations: at distances shorter than the average spacing between bosons one expects the behavior typical of an insulator, i.e. a constant. A reasonable form for the crossover

between short-distance insulating behavior and the asymptotics of eq.(3) then is[23]

$$\langle n_r n_0 \rangle \approx \cos(4k_F r) (\ell^2 + r^2)^{-K_\rho}, \quad (7)$$

where $\ell \propto 1/(1-n)$ is a length proportional to the average distance between bosons. On the other hand, the asymptotics (2) of the pairing correlations are expected to be valid only at distances larger than ℓ , with exponential decay at shorter distances. This in particular implies that the amplitude of the divergence of the pairing susceptibility ($\chi_{SCd} \propto \max(\omega, q, T)^{-3/2}$ as $K_\rho \rightarrow 1$) vanishes as $n \rightarrow 1$.

Recently, Konik *et al.*[11] have investigated the vicinity of the metal-insulator transition in the weak-coupling limit, starting from the $SO(8)$ model of the insulating state[10] and using the exact integrability of that model. They arrive at the same conclusion, $K_\rho = 1 + O(1-n)$. This provides a confirmation of the continuity between weak and strong correlation conjectured before.[6]

I finally comment on some properties of the model (5) for general fermion density. First, in the very dilute limit, corresponding to a dense bosonic model (6), from Haldane's results one again has $K_\rho = 1 + O(n)$. More interestingly, for a quarter-filled fermionic model (one boson per two sites), there is a metal-insulator transition into an insulating CDW state with period $4k_F$ when $V_{\text{eff}} > 2t_{\text{eff}}$. In the XXZ spin chain, this transition is due to umklapp processes [22] which in the original fermionic language of eq.(5) correspond to four-fermion umklapp processes, as first discussed in ref.[23], and which can become relevant only for strongly repulsive interactions. In the conducting state ($V_{\text{eff}} < 2t_{\text{eff}}$) one has $K_\rho \geq 1/2$, i.e. superconductivity dominates. On the other hand, for $V_{\text{eff}} > 2t_{\text{eff}}$, when $n \rightarrow 1/2^\pm$ one has $K_\rho \rightarrow 1/4$, i.e. $4k_F$ CDW fluctuations dominate. The $t - J - J_\perp$ model thus provides an interesting example for the transition between dominant superconducting and CDW fluctuations. Finally, we remark that in the case of strongly attractive interactions, $V_{\text{eff}} < -2t_{\text{eff}}$, one has phase separation for any filling of the band.

In conclusion, based on a solvable limit of the $t - J$ ladder model and a continuity conjecture between strong and weak correlation, I have shown here that close to the doping-induced metal-insulator transition the Luttinger liquid parameter K_ρ of the two-chain model *takes the universal value unity*, corresponding to dominant d-type superconducting correlations. This contrasts strongly with the single-chain case, where $K_\rho \rightarrow 1/2$ and antiferromagnetism dominates. It should however be pointed out that superconductivity is rather easily destroyed by disorder,[24] and that therefore in any real system the existence or not of superconductivity will depend crucially on the interplay between disorder and interladder coupling which stabilizes superconductivity.

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